FST 6-1 Notes
Topic: Introduction to probability
GOAL
Introduce the basic vocabulary and principles of probability. Calculate probability for events in which the sample space is small and finite and the outcomes are equally likely.

SPUR Objectives

A Compute probabilities of events in various contexts.
G List sample spaces and events for experiments.

Vocabulary

probability theory
experiment
outcome
sample space
event
probability of an event, $P(E)$
fair, unbiased
randomly, at random
empty set, null set

Experiment	Sample Space
flipping a coin	{heads, tails}
tossing a six-sided die	{1, 2, 3, 4, 5 6}
taking an antibiotic for a sore throat	{sore throat cured, sore throat continues}
picking an integer from 1 to 100	$\{n \in \mathbb{Z} \mid 1 \le n \le 100\}$

WARM UP
A drawer contains r red socks, b blue socks, and w white socks. Assume that you draw a sock randomly from the drawer.

1. What is the probability that it is red? $\frac{r}{r+b+w}$
2. What is the probability that it is not red? $\frac{b+w}{r+b+w}$
3. What is the probability that it is green? $\frac{0}{r+b+w} = 0$

Term	**Definition**	**From Warm up**
Experiment	A situation with several possible results	picking a sock from a drawer
outcome	the result of an experiment	the color of sock that was picked
Sample space	set of all possible outcomes	all the socks in the drawer
event	desired outcome, subset of sample space	in part a, the event was a red sock

Example 1: A small box contains 30 blue, 30 green, and 25 red paper clips. Two paper clips are taken from the box and their colors are recorded.

a) List all possible outcomes for this experiment.

BB, BG, BR, RR, GG, RG

b) How many outcomes are in the sample space for this experiment?

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Example 2: Two fair dice are rolled, both 6-sided.

a) What are all the possible outcomes?

{(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)} = 36

b) List the outcomes if the event is a sum of the number is 5.

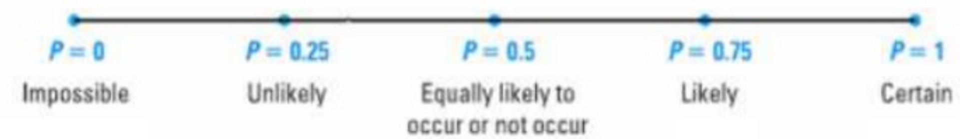
{(1,4) (2,3) (3,2) (4,1)}

c) Give the probability that the sum of the numbers is 5.

$\frac{4}{36}$

The probability of an event is a number from 0 to 1 that measures the likelihood or chance that the event will occur. It can be written as a fraction, decimal or percent.

"Equally likely" means each outcome has an equal chance of happening.



Definition of Probability of an Event

Let E be an event in a finite sample space S. Let $N(E)$ and $N(S)$ be the numbers of elements in E and S, respectively. If each outcome in S is equally likely, then the probability that E occurs, called the **probability of E** and denoted $\mathbf{P(E)}$, is given by

$$P(E) = \frac{N(E)}{N(S)} = \frac{\text{number of outcomes in event}}{\text{number of outcomes in the sample space}}.$$

$$prob = \frac{\text{\# desired outcomes}}{\text{\# possible outcomes}}$$

$30+30+25=85$ Total

30 blue, 30 Green, 25 red

Example 3: In the situation of Example 1, which of the six outcomes are equally likely? What are their probabilities?

$BB = \frac{30}{85} \times \frac{29}{84} = \frac{870}{7140}$ $\quad GG = \frac{30}{85} \times \frac{29}{84} = \frac{870}{7140}$

$BR = \frac{30}{85} \times \frac{25}{84} = \frac{750}{7140}$ $\quad GR = \frac{30}{85} \times \frac{25}{84} = \frac{750}{7140}$

$BG = \frac{30}{85} \times \frac{30}{84} = \frac{900}{7140}$ $\quad$ BB+GG

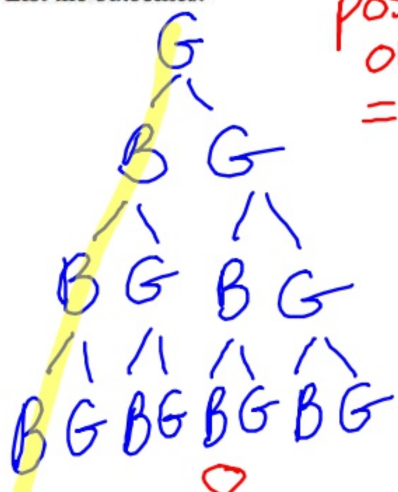
$RR = \frac{25}{85} \times \frac{24}{84} = \frac{600}{7140}$ $\quad$ BR+GR

Example 4: Assume that births of boys and girls are equally likely. A family has 4 children.

a) How many outcomes are in the same sample space? List the outcomes.

possible outcomes = 16

8



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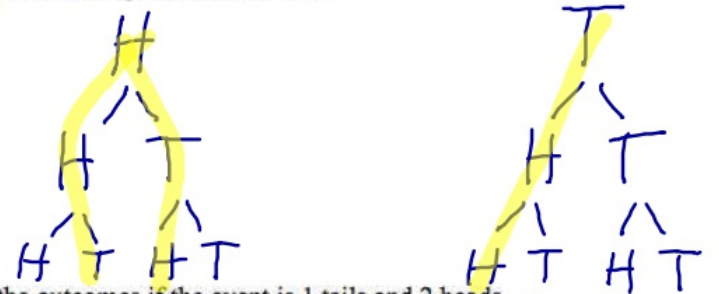
b) Find the probability that a family of 4 children has one girl and three boys.

BBBG, BBGB, BGBB, GBBB $= \frac{4}{16}$

Example 5: Three fair coins are flipped.

a) What are all the possible outcomes?



Possible outcomes = 8

b) List the outcomes if the event is 1 tails and 2 heads.

{HHT, HTH, THH} = 3

c) Give the probability for the event in part (b).

$$\frac{3}{8}$$

If an event contains no possible outcomes, then it cannot occur and it has probability of 0. A set with no elements is called the **empty set** or the **null set** and is denoted either by the symbol $\{\ \}$ *or* $\varnothing$. $P(\{\ \})$ *or* $P(\varnothing) = 0$.

If an event is certain to happen, then it contains all the possible outcomes in the sample space, and has probability 1. $N(E) = N(S)$, *so* $P(E) = \dfrac{N(E)}{N(S)} = 1$.

Theorem (Basic Properties of Probability)

Let S be the sample space associated with an experiment. Then, for any outcome or event E in S,

(i) $0 \le P(E) \le 1$.
(ii) if $E = S$, then $P(E) = 1$.
(iii) if $E = \varnothing$, then $P(E) = 0$.

Relative Frequencies and Probabilities

- They are related, but their meanings differ
- Both have values that range from 0 to 1
- A relative frequency of 0 means the event has not occurred, but this does not guarantee that the probability is 0 (same if relative frequency is 1)
- However, if probability of an event is 0, then the relative frequency is also 0 (same if probability is 1)